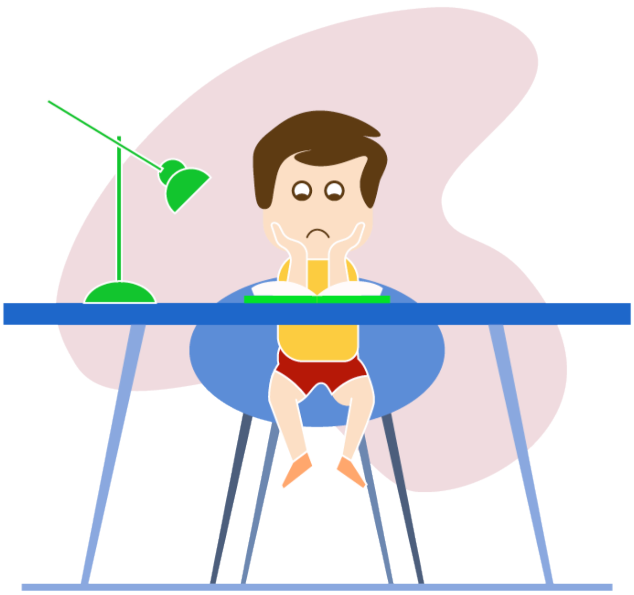
Tables Logik

How to help your child learn their times tables.

STUDY SMART



Perhaps we firstly consider how NOT to learn your times tables. Many learners have found to their frustration that merely trying to memorise a number sentence written out only in the abstract form (ie numbers & symbols) can be difficult. Not least because there is no meaning attached to the facts they are attempting to learn. Even with a variety of games, rhymes or other motivational strategies, learners can not only struggle to commit to memory the basic fact but they also don't make the connections to other related facts.

For example learning just 4 x 6 = 24 doesn't necessarily make the connection to the associated family of facts i.e.

6 x 4 = 24, 24 ÷ 4 = 6 and 24 ÷ 6 = 4

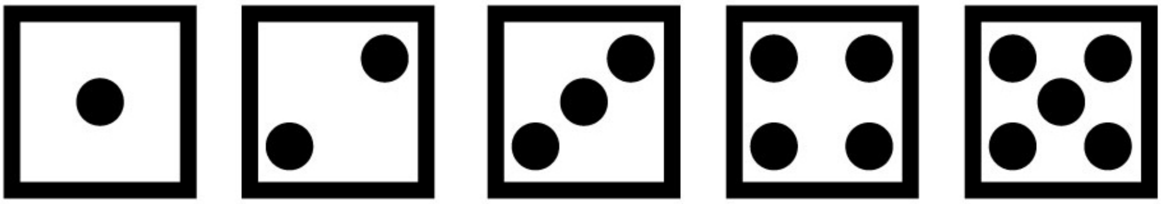
and even less likely to make the link to other associated facts such as

2 x 12 = 24 or 8 x 3 = 24.

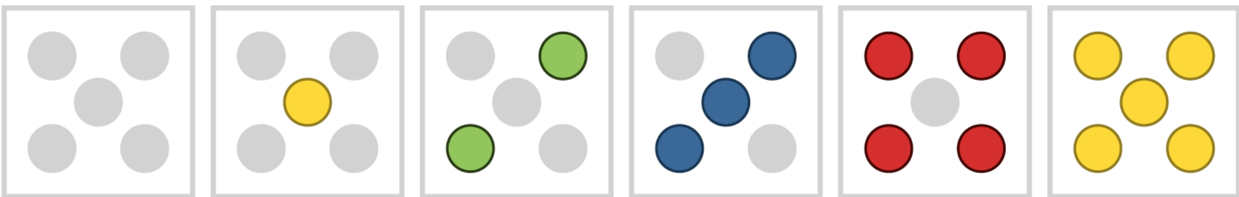
The problem with the traditional method is the abstract nature of the fact being learned. What the Logik method proposes is to make use of a visual representation of the fact. So, when you are asked to remember what two fours is, the learner actually visualises (at least during the learning process) what two fours looks like.

Visualising a number

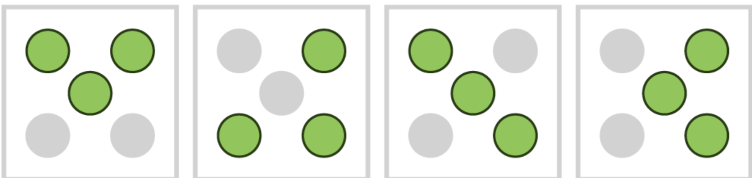
In order to use the Logik strategy, learners need to have a visual representation of the numbers 0 to 5 to work with. The most common representations used are the typical representations on the side of a dice or die.



However, there isn't usually a zero represented on a die. So rather than just an empty box Tables Logik uses the faint outlines of the missing counters. This also has the benefit of learners "seeing" the complement to five of the digit represented and later the complement to ten also becomes apparent.



It is important to note these representations aren't necessarily fixed, so for example a three can be represented in many different ways, for example;



Each can satisfactorily represent three and additionally show that three is "two less" than five.

Subitising

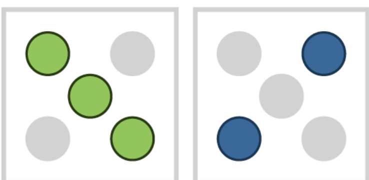
The use of these simple representations builds upon the early development of number sense and the innate subitising skill of young learners. Once the representations of the numbers 0 to 5 are established the beginnings of times tables facts can be introduced. The simplest times tables facts will be learned alongside (and as a consequence of) the learning of addition number bonds facts. To begin with a learner will probably not consider these as times table facts per se but as simple addition facts.

Whilst 5 + 5 = 10 and 2 x 5 = 10, formal times tables are developed over time

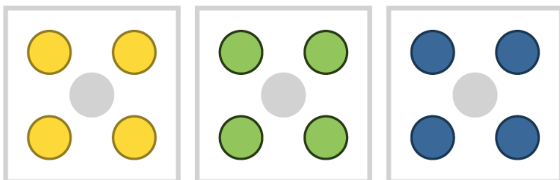
It is important at this stage for us to distinguish between developing young learners' initial understanding of number and helping older learners to grapple with times tables after they have a reasonably well developed number sense. For the remainder of this introductory article, the strategy described is for the latter, older learner.

Visualising number facts.

A key feature of Tables Logik is the visualisation of the number fact represented by a number sentence. This can be a simple addition fact such as 3 + 2 = 5 or a multiplication fact such as 3 x 4 =12.



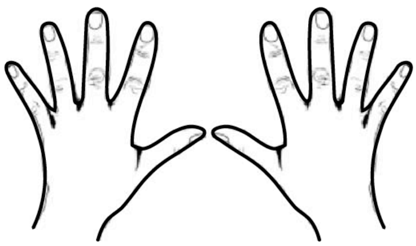
3 + 2



Three four´s

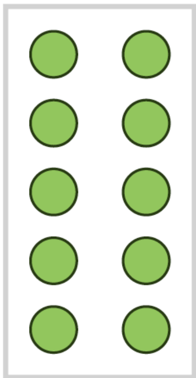
Ten is made up of two fives.

Another important aspect for learners to be comfortable with is the notion of a ten being made up of two fives. This is a key feature of the Logik strategy of developing number sense. But for those learners who have not met this before it is a natural development of their understanding of number from using the fingers on two hands.



1

Ten as two hands



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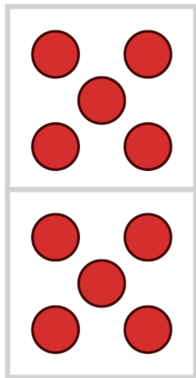
Numicom tens frame



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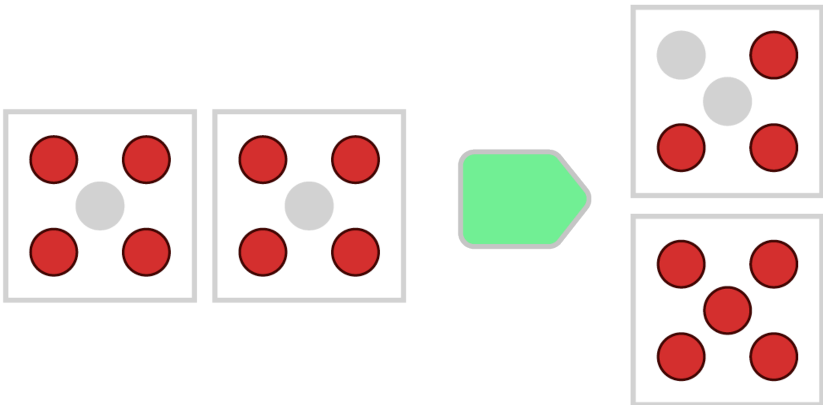
Hungarian tens rame

Establishing a fact using "concrete" manipulatives.

Much is written in educational theory of the benefit of using manipulatives in the learning process. The Logik strategy encourages learners to use counters to explore a new fact through actual manipulation of counters which in turn will support their later visualisation and mental agility.

Let's look at some examples;

Two fours or 2 x 4.



A learner presented with two fours can explore moving the counters onto a tens frame to establish that 2 x 4 = 8. The learner should be asked how they know that two fours equals eight and the answers will reveal a developing sense of equality built upon their earlier addition and number bonds fluency. In time they will be able to explain through visualisation and memory only.

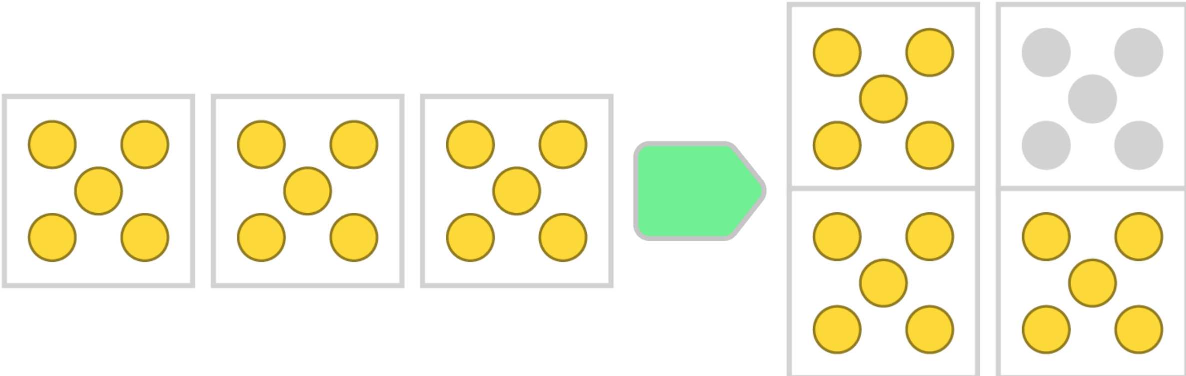
What numbers can you see?

Can you move the counters to show the total is eight?

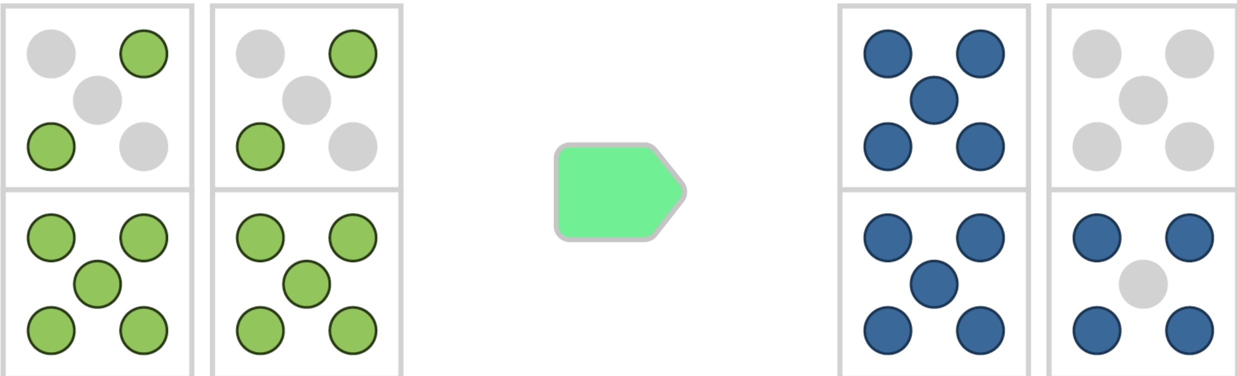
Can we write this as a number sentence?

Three fives or 3 x 5.

This example uses the grouping of fives into tens and therefore no actual movement of counters may be necessary but by talking through their reasoning learners begin to develop important strategies for themselves and consolidate their understanding of how the number system works.



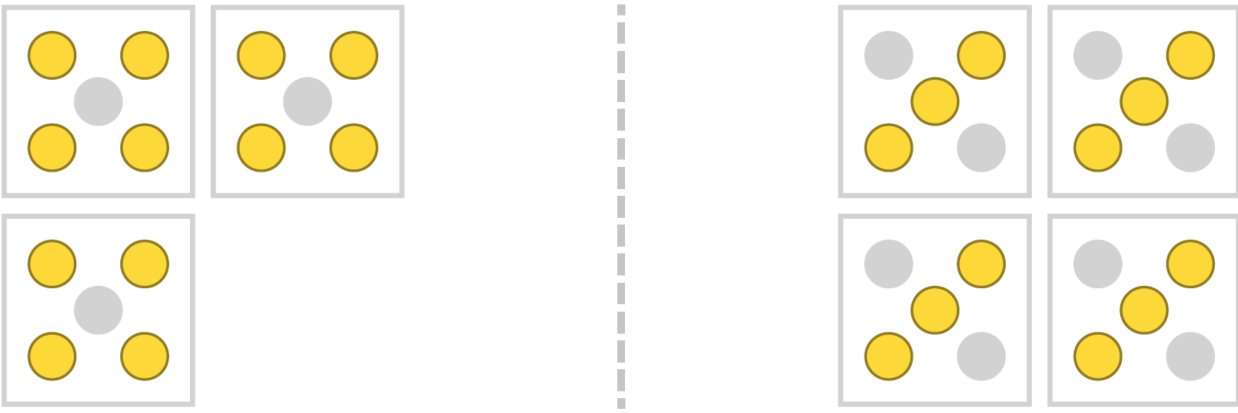
Two sevens or 2 x 7.



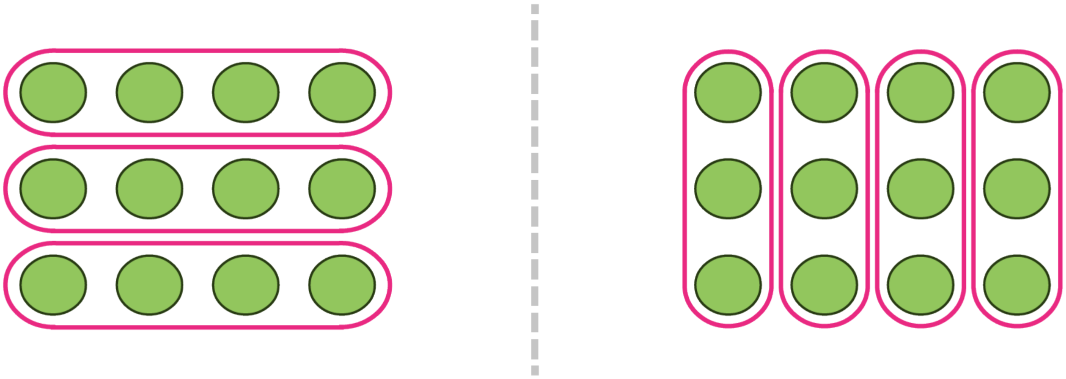
This is an important example that illustrates how learners may move the counters to establish the answer on a tens frame but also use logic and reasoning to explain where they "see" the ten being made up of two fives and the four made up of two twos. This is an important step towards using the distributive law more widely in establishing more complex facts later.

Knowing & understanding commutativity

Up until now this chapter has assumed a multiplication fact such as 3 x 4 is interpreted as three fours but we know that in fact four threes is an equally correct interpretation and whilst three fours and four threes are distinctly different, the product of each of these multiplications is the same.



Four threes and three fours are distinctly different but their product ie 12 is the same.

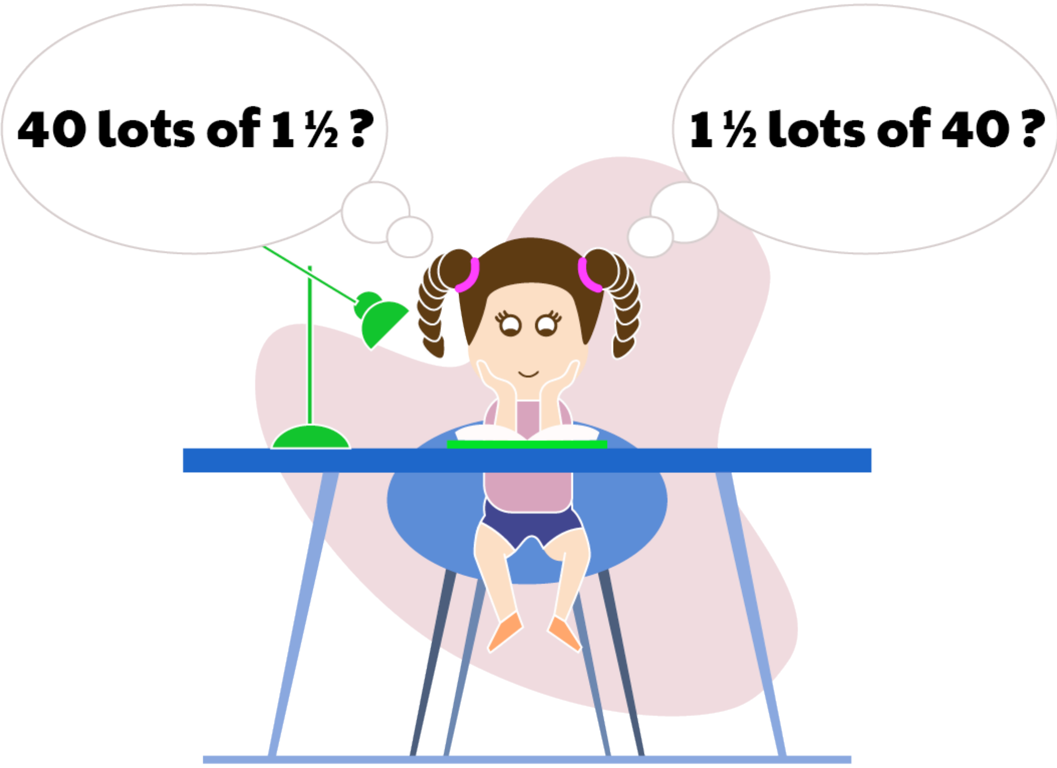


Three fours Four threes

Learners can verify that the product of both is twelve by rearranging counters and they should also be introduced to the rectangular array arrangement of counters which can illustrate the commutativity of two-number multiplication facts by considering either the rows or columns in each case.

Multiplication of two numbers can always be interpreted in two different ways but the product is the same, so learners should be encouraged to calculate the answer in the way the find they find the easiest

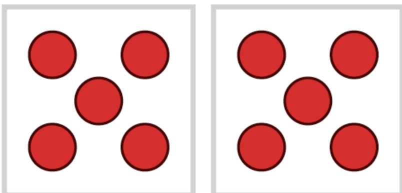
This is a strategy they will continue to use beyond the standard times tables and even into multiplication of fractions and decimals.



Choosing what works best for you.

Discussing the illustration of a number fact with learners helps them arrive at what makes best sense for them to remember. For example five twos and two fives can be made with counters and the learner can decide which is the easiest for them to remember that the product of both of these is ten.

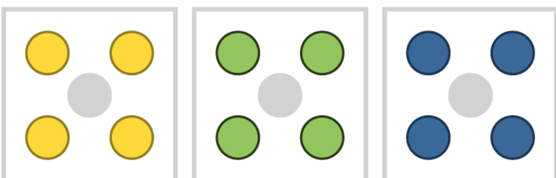
Five twos, or two fives ?



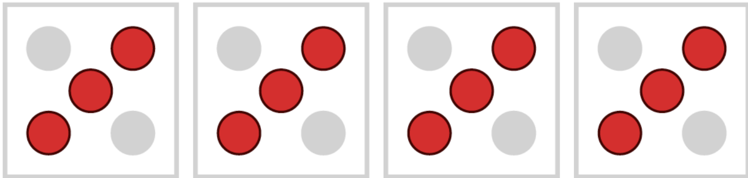
Which is the best way for you to remember 2 x 5 = 5 x 2 = 10 ?

The best way is your way.

In the case of 3 x 4 , learners could choose one of any number of different ways to describe how they can best remember the product is 12. There is no “right answer”, for example:



Some may describe the movement of two counters from one four to make two fives (ie a ten) out of the other two fours, leaving two and so ten & two makes twelve.



Others may “see” four threes as two sixes, knowing that a six is 5 + 1, so two sixes is 10 + 2 ie 12.

The important aspect here is that the learner themselves choose the way they feel most comfortable with to explain the answer and so use this to remember until after practice (which is always essential) they begin to answer automatically and fluentl .

Searching for different methods.

So when learning a new times tables fact learners should be encouraged to explore different ways they can “prove” the fact to be true. This gives them an opportunity to try out different explanations before settling upon the one they want to choose to help them remember. It also allows them to make links with other number facts and strategies. For example,

Two eights or 2 x 8.

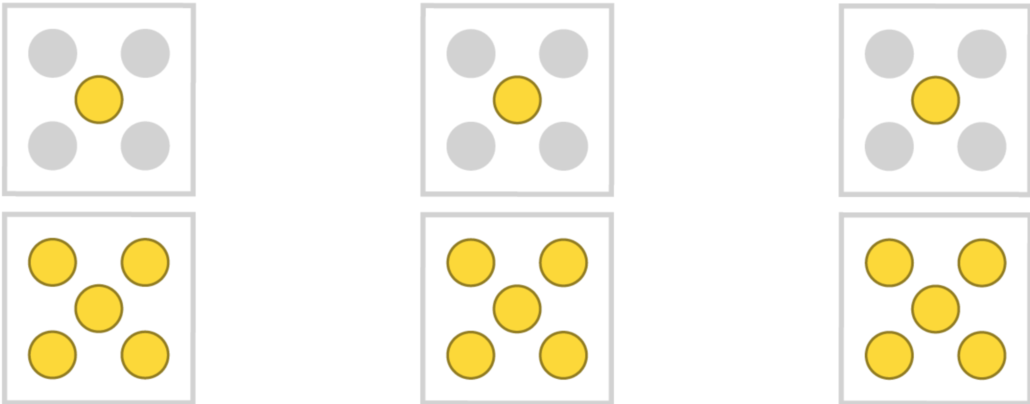
Some may see the two fives as a ten and the two threes as six and so combine these answers to make sixteen.

Alternatively some learners may choose to explain that eight is “two less” than ten and so two eights will be four less than two tens (or twenty) which is also sixteen.

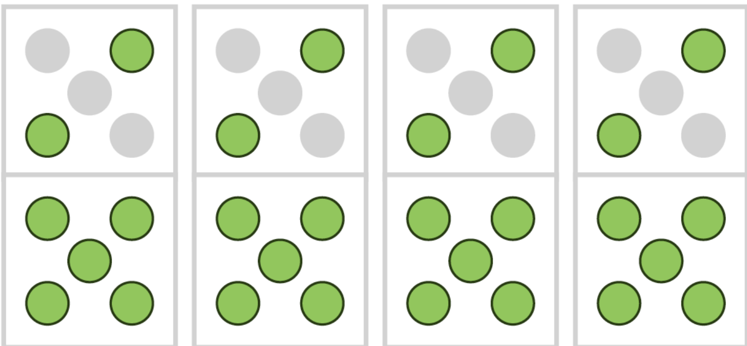
Common methods.

This use of the distributive law by partitioning a number into two parts (for example eight into five and three) is a common and useful strategy.

a) The distributive law and "just above five"



Sixes can be partitioned into fives and ones so three sixes can be thought of as three fives and three ones (i.e. 15 + 3) and so the answer is eighteen.



Similarly sevens can be partitioned into fives and twos, so four sevens would be the same as four fives (20) and four twos (8) hence the product is twenty-eight.

It is clear that this method relies on the fluency of previously learned facts but consolidating their knowledge in this way not only practices these earlier learned facts but also develops mental agility and strategies that can be applied elsewhere.

b) The distributive law and "just above" and "just below" ten

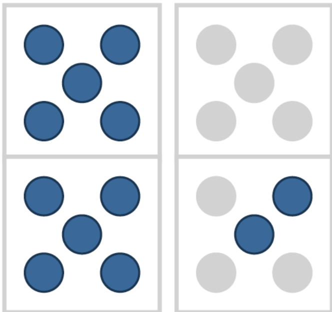
Too often learners are introduced to tricks for the eleven and nine times table, but without the reasoning behind them which could be applied to other numbers. Let's consider two examples; two nines and two elevens

In the case of two nines, the nines can be thought of as "one less than ten" and so two nines will be two less than twenty ie eighteen. This same idea can be applied to the whole of the nines times table.

Similarly for two elevens, the elevens ca be thought of as "one more than ten" and so two elevens will be two more than twenty ie twenty-two. Again this idea can be applied to the whole of the elevens times table.

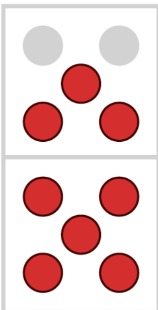
Neither of the above may ensure that learners recall the nine or elevens times tables better than by learning the "trick" methods BUT those same strategies can now be applied to eights and twelves and even to thirteen.

For example consider four twelves as four tens and four twos.



Can you explain why 4 x 12 = 48 ?

Similarly consider four eights as four tens missing four twos.



Can you explain why 4 x 8 = 32 ?

What have we established so far?

a) Don't just expect learners to memorise numbers & symbols

b) Learn new facts by exploring them with counters & tens frames

c) Ask learners to explain how they know a fact to be true

d) Remind learners that for each tables fact, A lots of B is equal to B lots of A

e) Ask learners to think of different ways of proving the fact and get them to choose the one that makes the most sense to them

f) Encourage learners to remember by visualising their chosen explanation

g) With practice learners will begin to answer immediately without the explanation

Added bonus

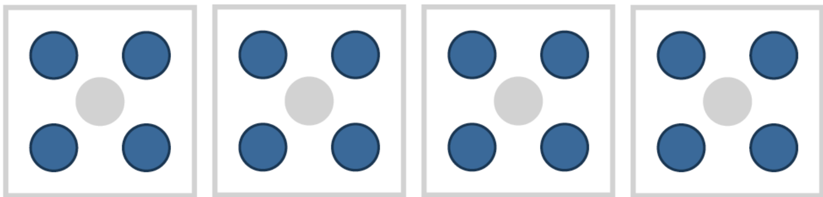
- Learners will grow in confidence particularly in explaining their methods
- Learners will develop new strategies for mental arithmetic
- These new strategies will be useful for more complex calculations

Grouping and using earlier known facts.

Whilst learners may regularly use the "just above five" or the "just above/below ten" methods for calculating there is another type of common method to be aware of.

This strategy uses a simpler fact to calculate an equivalent or multiple.

Consider four fours,



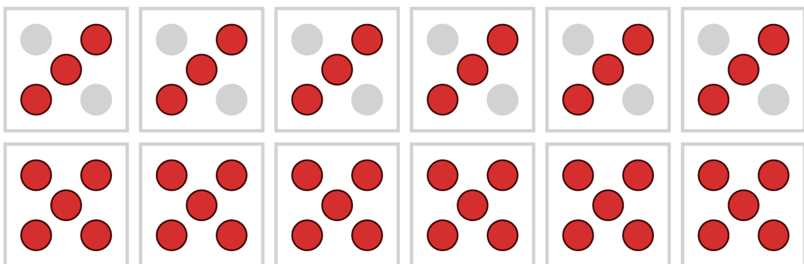
Some leaners may recognise this illustration as being equal to two eights, and if they already know two eights to be equal to sixteen then four fours must also be sixteen.

Another similar example would be four threes and two sixes, can you visualise this?

But the same strategy can be useful for calculations beyond the standard times table, for example three fourteens could be shown to be equivalent to six sevens.

Multiples.

A similar method can be used for example when calculating six eights.



If a learner already knows that three eights equals twenty-four they may use this to show that six eights would be double this answer i.e. forty-eight.

However learners should still be encouraged to look for alternative ways, Six eights is the same as six fives and six threes i.e. 30 + 18 = 48

OR

Six eights is also the same as six tens minus six twos i.e. 60 – 12 = 48

OR

Eight sixes is the same as eight fives and eight ones i.e. 40 + 8 = 48

choose your favourite!

Practice and tracking progress

Whilst the emphasis of learning the facts using the Logik strategy is on "seeing" the fact and understanding how it works, the aim is for learners to be able to fluently and rapidly recall these facts without the need for visualisation and explanation.

There are important aspects to consider when practising.

a. The order of facts to be learned
b. The level of support provided
c. The level of response expected

Order of facts to be learned

Here is a suggested order of times tables facts but this may well be different for individual learners as their previous experience may be varied.

Earliest stage:

- Any number multiplied by one or zero
- Numbers 0 to 4 multiplied by 2
- Easier multiples of fives and ten

Simple tables:

- Numbers 0 to 4 multiplied by 3,
- Numbers 6 to 9 multiplied by 2
- Further multiples of fives & ten

Developing tables knowledge

- Numbers 0 to 4 multiplied by 4
- Numbers 6 to 9 multiplied by 3
- Confident use of fives & tens tabl
- Introducing simpler 11 & 12's tables (e.g. multiplied by 2 to 4)

Building on what you know

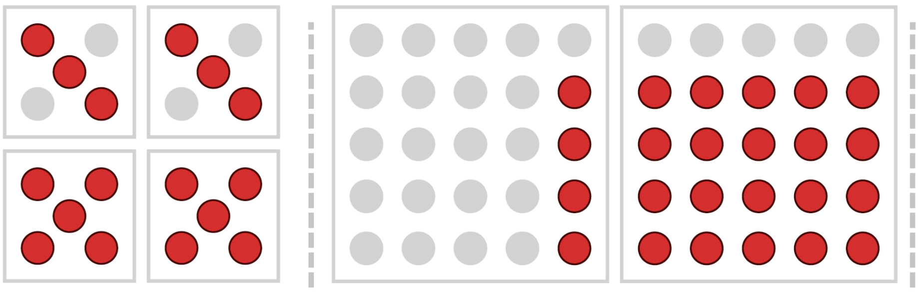
- Numbers 6 to 9 multiplied by 4
- 9 & 11's tables multiplied by numbers up to 10
- 12' tables multiplied by numbers up to 5

Filling the gaps

- Remainder of 6 times tables
- 6 x 7, 7 x 7, 8 x 8, 11 x 11
- Remainder of 12 times tables

Level of support provided

- Pictures of the fact AND the symbolic expression
- Symbolic expression AND the associated array
- No illustrations, symbolic expression only



8 x 2 = 2 x 8 = ? | **4 x 6 or 6 x 4 = ?** | **5 x 7 = ?**

Level of response expected

- Answer with explanation, no time limit
- Answer only
- Answer only with time limit
- Mixture of product answer or either of the multipliers with time limit

Learners will go through stages towards fluency and so they should be expected to give an appropriate level of response. As they begin to learn a fact they should be encouraged to explain how they know, but as this knowledge develops a quicker response should be encouraged to the point that this becomes a rapid recall.

Developing fluency would be evident once the learner can respond not only with the product of the multiplication but also if given the product and one of the multipliers can rapidly respond with the other multiplying number.

Tables Logik

Summarising the process

#1.

Use counters to explore a new fact

#2.

Talk about different ways you can show the fact is true and pick your favourite method

#3.

Practice by explaining your chosen method each time

#4.

Practice by visualising your chosen method

#5.

Practice to get faster

#6.

Remember fluency is more than just remembering the answer, so practice too by recalling missing factors as well as the product.